**Game 8: Puzzled by Projection II**

**Why?**

In Game 7, we looked at how projections can capture one-sided views of an object and predicted projections given a model and an angle. Now, it is time to go at it the opposite way: to figure out what’s inside an object by requesting projections of your choice! Indeed, this is essentially what we need to do when presented with raw MRI or CT data. In this game, you will be probing a mystery 3D object or 2D image with a limited number of 2D / 1D projections.

**Materials**

* Calibration tube
* Mystery tubes

**Background**

1. Key words

* Forward and inverse processes
* Encoding
* Reconstruction

1. Basics

This game will task you with figuring out the inner structure of a 3D model by performing and examining just a few projections. Are you up to the challenge?

1. Explanations

Forward and inverse processes

In daily life, we sometimes encounter pairs of *forward* and *inverse* processes. For example, a puzzle can be made by cutting a single picture into many pieces, which is a forward process. The re-assembly of the pieces would be the inverse process. Some processes are hard to invert: for example, a single 1D projection curve (explored in Game 7) cannot be used to determine for sure what the original 2D image was. However, if we combine 1D projections from many angles (like individual puzzle pieces!), we might be able to recover the image. In MRI, this inverse process is called reconstruction, whereas the initial forward process is called encoding. MR imaging involves many forward processes that need to be inverted so we can see the whole picture. We may not be able to perform the inversion perfectly but the goal is to have a good-enough estimate of the original image.

Encoding

We always encode information about space in MRI. That is, we need to tell where in 2D or 3D space a signal belongs so we can make an image by assigning a signal to each position. There are two ways of encoding space: Cartesian and non-Cartesian. In MRI, spatial encoding is the same as sampling from k-space (see Game 2), which has the same size as the image. Cartesian encoding makes sure we can sample k-space on a grid (think of a dotted notebook or wire crossings of a cooling rack, left figure). Non-cartesian is more general and includes all types of sampling that do not fall on a grid (right figure).

A diagram of a sampling

Description automatically generated with medium confidenceA diagram of a radial sampling

Description automatically generated

Reconstruction

* Reconstruction from Cartesian data is easy: take an Inverse Fast Fourier Transform (IFFT, the inverse process in Game 2), which can be done easily with code.
* Reconstruction from non-Cartesian data is trickier. There are many methods. Two of them are:
  + Backprojection: this only works for radially sampled data and is done in two steps: (1) Take a 1D Fourier transform of each radial line, which gives you *a 1D projection in the corresponding direction* (thanks to a mathematical fact called the Projection Slice theorem); (2) Perform a procedure called backprojection with this set of 1D spatial projections to get back the image. The figure below shows how backprojection works:

A diagram of a graph

Description automatically generated

(<https://www.dspguide.com/ch25/5.htm>)

What we do is to take each projection and smear it across the image in the correct angle. The intersection of the projections highlights the location of the object and gets more accurate as more projections are included. Backprojection is also the basis for reconstructing CT (CAT scans) images!

* + Gridding: First, we use the non-cartesian data to make an estimate of the cartesian data. Then we use the regular IFFT on the estimated cartesian data to reconstruct the image.

**Lab procedures**

In this game, instead of going over the math of these methods in detail, we would like to hone your intuition by asking you to “reconstruct” these images/volumes **just with your brain**. This way, you can appreciate that all the information we need to *make an image* or *tell several images apart* may be contained in one or more of its projections.

1. 2D puzzle
2. Select “2D” on the left panel to enter 2D puzzle mode. A mystery image shows up! The “options” panel on the right shows multiple possible images to choose from.
3. Your job is to choose projection angles to view the image from. Enter an angle in the ‘1D proj. angle” space and press “IMAGE 1D” to get your first projection.
4. Your first projection is now visible in the middle panel, on the first tab. Keep changing the angle and pressing “IMAGE 1D” to get up to five projections.
5. Using all these projections, decide which of the options is the correct image (which one is consistent with all the projections you got?). Then choose one of the options under “Answer” and press “SUBMIT” to check your answer.
6. Whether you got it right or wrong, you can press “NEW MODEL” to proceed to the next mystery image!
7. 3D puzzle
   1. Select “3D” on the left panel to enter 3D puzzle mode. A mystery object shows up! The “options” panel on the right shows multiple possible objects to choose from.
   2. Your job is to choose projection angles to view the image from. Choose among “x”, “y” and “z” in the ‘2D proj. axis” field and press “IMAGE 2D” to get your first projection.
   3. Your first projection is now visible in the middle panel, on the first tab. You can now choose one more different axis to get up to two projections.
   4. Using the 1-2 projections, decide which of the options is the correct object (which one is consistent with all the projections you got?). Then choose one of the options under “Answer” and press “SUBMIT” to check your answer.
   5. Whether you got it right or wrong, you can press “NEW MODEL” to proceed to the next mystery object!
8. Hints
   1. The angles and axes directions correspond to the ones in Game 7.
   2. Look over your choices before deciding which angles/axes can best tell them apart.
   3. Use “RESET” to keep the same image/object but get another chance at answering so you can find out the correct answer. If you are having trouble, go through a few questions in this way to understand why the projections look the way they are before proceeding without the help of “RESET”.

**Questions**

1. Which of the following is a pair of forward and inverse processes?
   1. Baking a cake and eating it
   2. Growing a tree from a seed and harvesting a seed from that tree
   3. Making a piece of art and selling it to buy more paint
   4. Putting together a LEGO house and taking it apart
2. Which of the following is not a way to sample k-space in a non-cartesian manner?
   1. Sample points with the same spacing across x and across y
   2. Sample points regularly along a line that crosses the center of k-space, rotate the line, and then sample it again
   3. Sample points randomly so more points are sampled from the center of k-space
   4. Sample points along a spiral
3. Which of the following is not a way to process and reconstruct raw MRI data?
   1. Performing an IFFT on it
   2. Convert 1D radial data lines to the spatial domain and perform backprojection
   3. Use gridding and then perform an IFFT
   4. Bake it at 450F for 40 minutes